Systems Analysis for Sustainable Development

Spring 2013
Monday, 18 Feb

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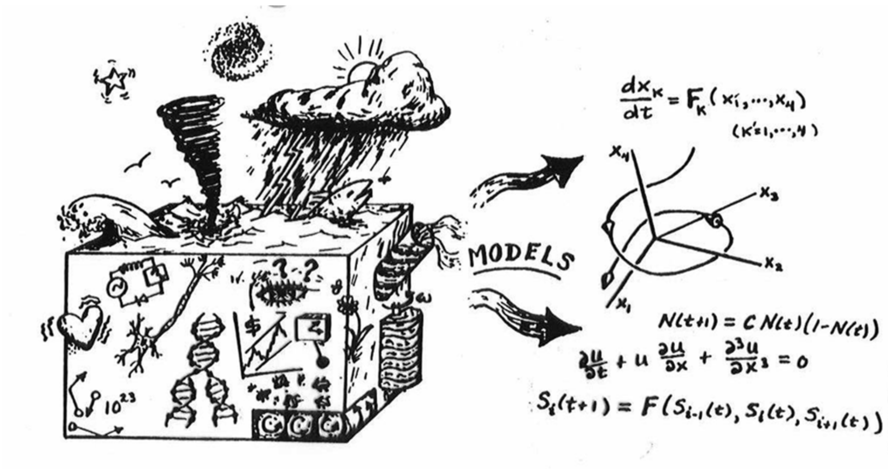
Today's session
Lecture 7

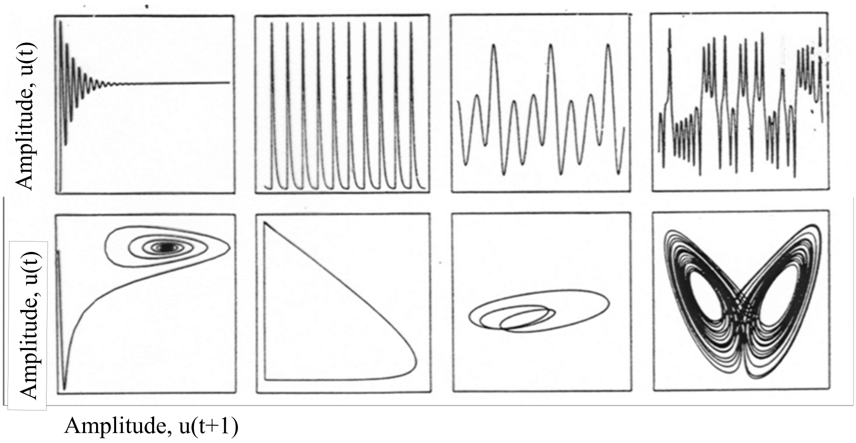
The wonderful world of simulations

Mathematical models of complex (natural) systems



Non-linear dynamics

Steady state oscillations chaos



Population dynamics

For the continuous change of population size, we have:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

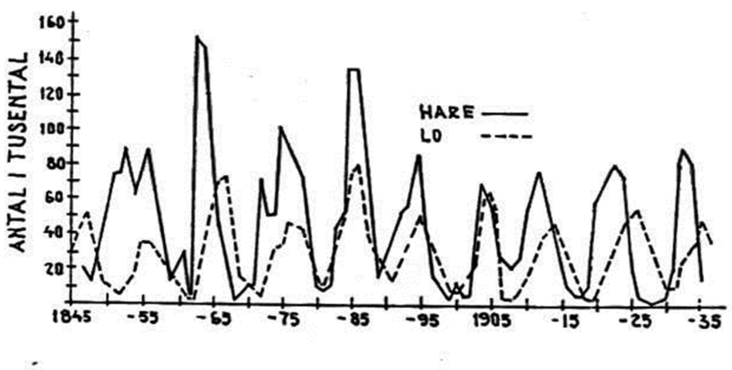
where

N = number of individuals in the population at time t

r = maximum rate of growth

K = carrying capacity of population, number of individuals at equilibrium

Dynamical balance between predator and prey



The Lotka-Volterra model

The Austrian mathematician A.J. Lotka (1880-1949) and the Italian mathematician V. Volterra (1860-1940) suggested a simple model for the way populations of predators and prey interact.

Let $x_1 = f(t)$ denote the predator population and $x_2 = g(t)$ denote the prey population at time t. The predator could be a foxes, and the prey could be a rabbits.

Lotka-Volterra model

For two species interacting in a predator-prey relation, we can set up a Lotka-Volterra model, where y_1 represents the prey and y_2 the predator:

$$\frac{dy_1}{dt} = y_1 - \alpha y_1 y_2$$

$$\frac{dy_2}{dt} = -y_2 + \beta y_1 y_2$$

Numerical solution of the L-V system

We solve the Lotka-Volterra model system numerically, with the aid of ODE23 and ODE45 in Matlab!

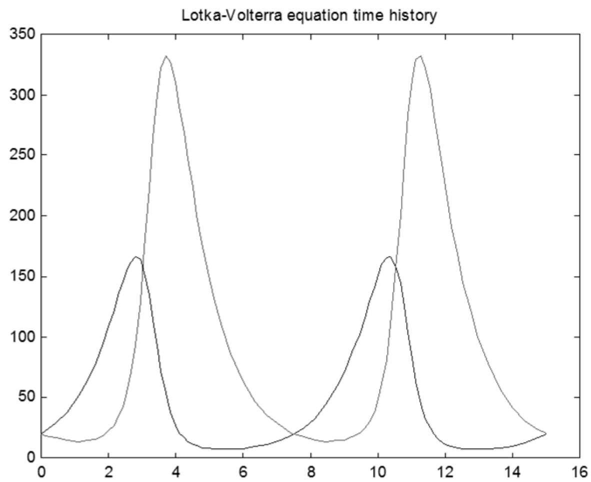
Lotka-Volterra system

```
%   To simulate the differential equation defined in LOTKA over the  interval  0 < t < 15, we invoke ODE23.  We will use the default  relative accuracy of 1e-3 (0.1 percent).

t0 = 0;

tfinal = 15;

y0 = [20 20]';   % Define initial conditions.

% [t,y] = ode23('lotka',[t0 tfinal],y0);

tfinal = tfinal*(1+eps);

[t,y] = ode23('lotka',[t0 tfinal],y0);

plot(t,y), title('Lotka-Volterra equation time history'), pause
```

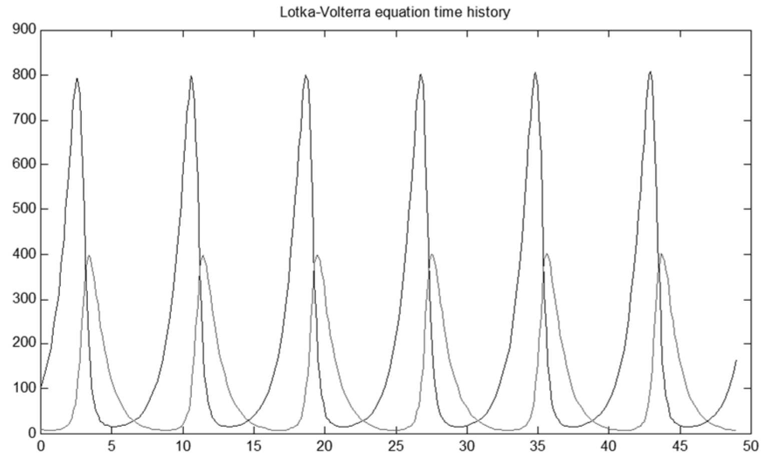
Lotka-Volterra model simulation

Matlab: lotkademoHL

Lotka-Volterra system



Model for predator-prey interaction



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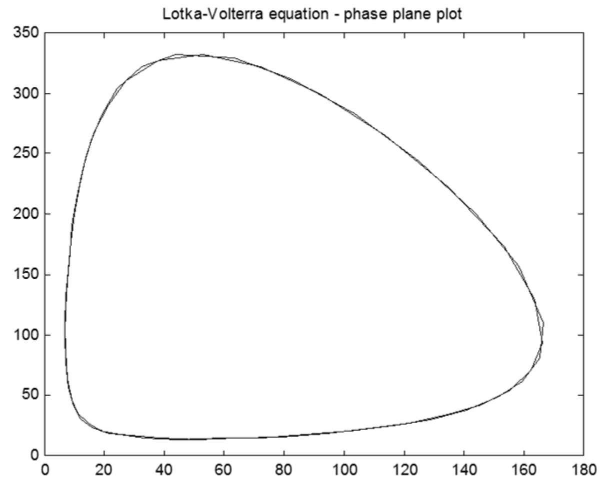
Lotka-Volterra system
For the predator-prey interaction



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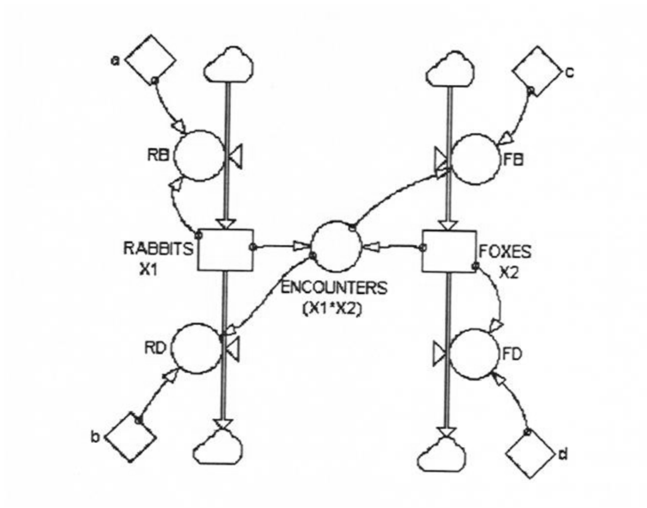
Lotka-Volterra model

In Powersim denotation:



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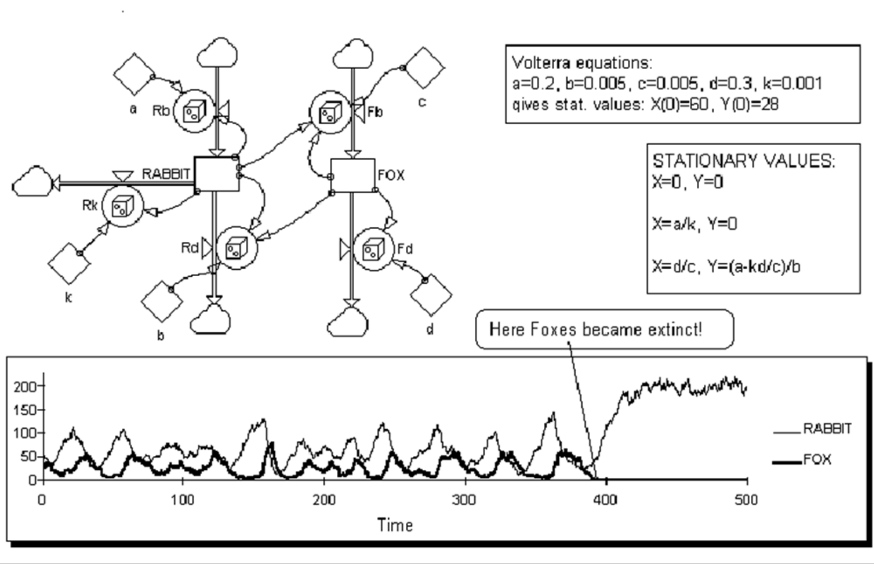
Lotka-Volterra model



Conclusions: 1) Stochastics excites dynamics! (Started in equilibrium!)
2) A stochastic model may switch to another dynamic ***mode***!

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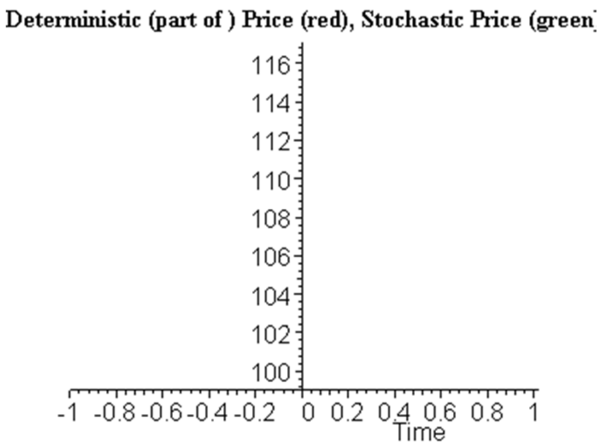
Why stochastic simulation?

Stochastics excites dynamics and dynamics change the stochastic conditions

- If modeled separately, both the statistic and the dynamic estimates will be wrong!
- Average from stochastic model may differ from that of a deterministic model.
- The model may switch between modes (cfr. Volterra).
- Deterministic models that behave exactly the same may behave quite differently when stochastics are added.
- Adds statistical estimates to a dynamic model.

Dynamics and stochastics must be treated together when both aspects are important !!!

Deterministic and stochastic processes



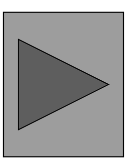
The Lorenz model

One of the most well studied system of differential equations with irregular (chaotic) behaviour was suggested by Lorenz as a reduced version of the coupling between convection and temperature transmission in a liquid or gas (the atmosphere):

$$\boxed{\begin{aligned} \frac{dx}{dt} &= -\sigma \cdot (x - y) \\ \frac{dy}{dt} &= rx - y - xz \\ \frac{dz}{dt} &= -bz + xy \end{aligned}}$$

x: convection flow
y: temperature transm
z: gradient temperature
σ: Prandlt number
r: Rayleigh number
b: a geometric constant

Lorenz attractor simulation

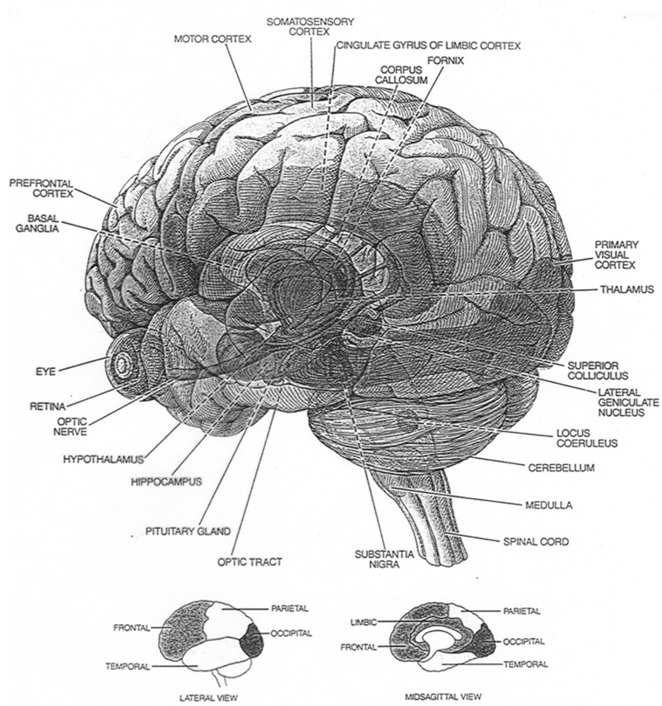


Matlab: lorenz

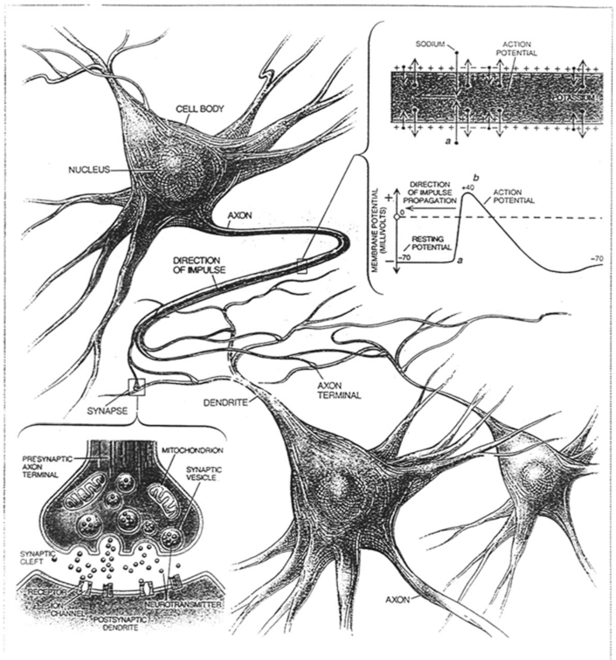
The Travelling Salesman Problem (TSP)

Matlab: travel

The Human Brain

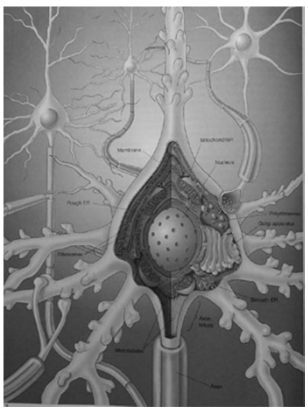


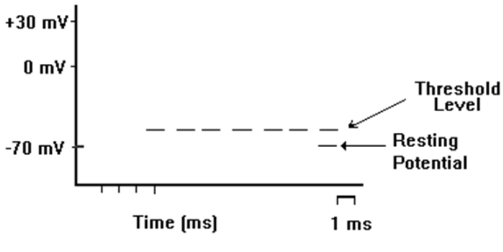
Neural commu-nication

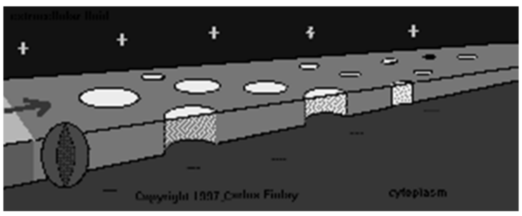


How Neurons Communicate

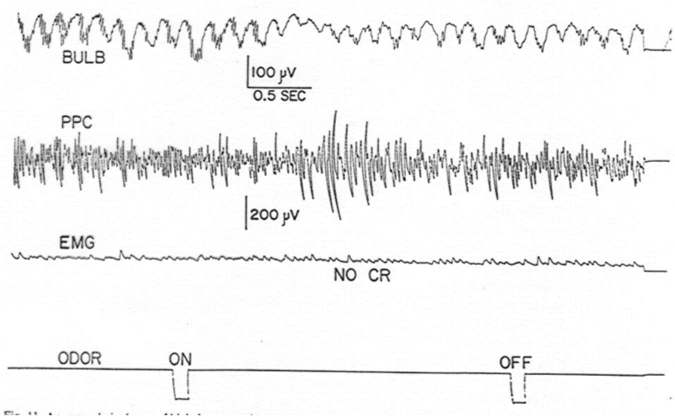
The functional unit process: The nervous cell impulse







Electrical activity of the brain
EEG from the olfactory bulb



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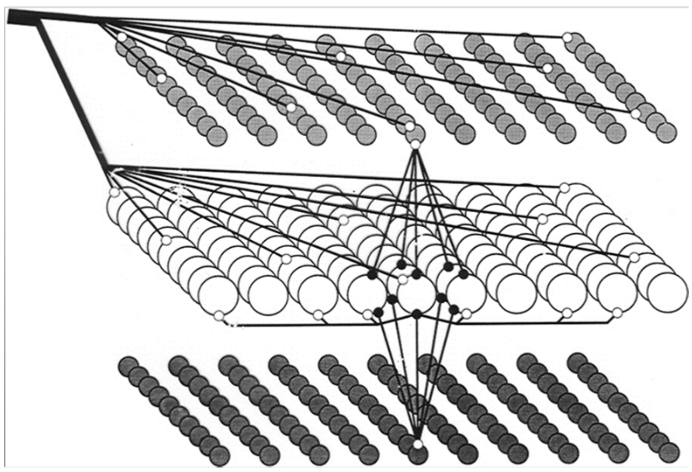
Neural network model of hippocampus



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Model equations

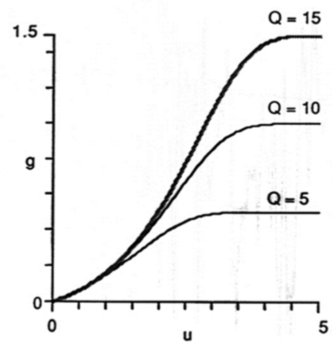
$$\frac{du_i}{dt} = -\frac{u_i}{\tau_i} + \underbrace{\sum_{j \neq i}^{N} w_{ij} g_j[u_j(t-\delta_{ij})]}_{\text{deterministic, possibly chaotic}} + I_i(t) + \underbrace{\xi(t)}_{\text{stochastic, noise}}$$

$$g_i = C \cdot Q_i \left\{ 1 - exp\left[-\frac{exp(u_i) - 1}{Q_i} \right] \right\}$$

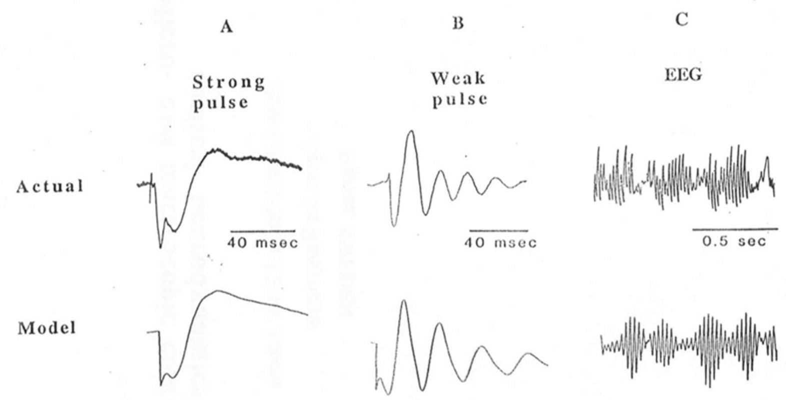
MODEL PROPERTIES

Experimentally determined input-output function (Walter Freeman, 1979)

$$g_i = C \cdot Q_i \left\{ 1 - \exp\left[-\frac{\exp(u_i) - 1}{Q_i} \right] \right\}$$



Simulation results



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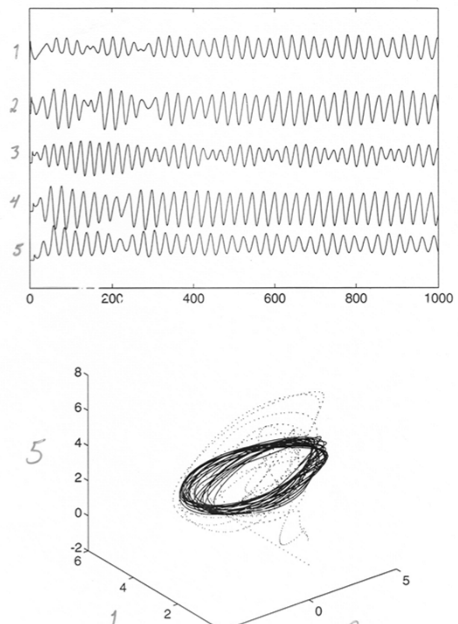
Time series and attractor dynamics



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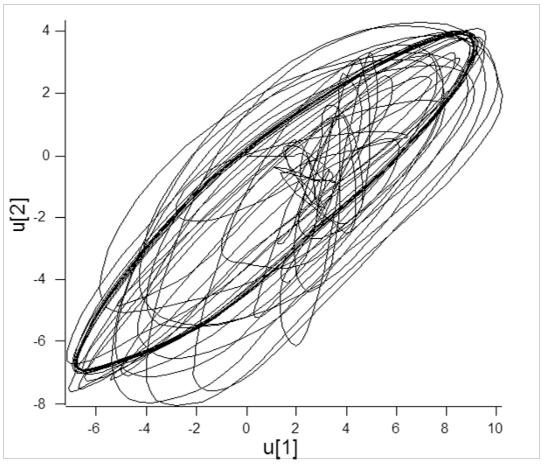
Memories are stored as (near) limit cycle attractors

After an initial period of chaotic-like dynamics, the system converges to a near limit cycle attractor



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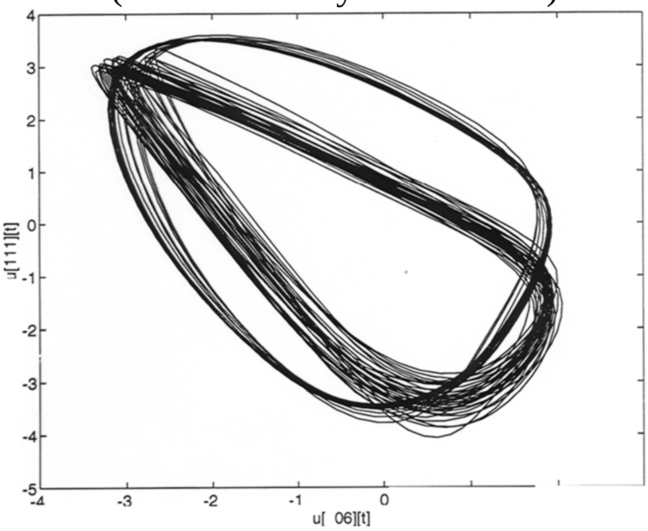
Several patterns can be stored

(as near-limit cycle attractors)



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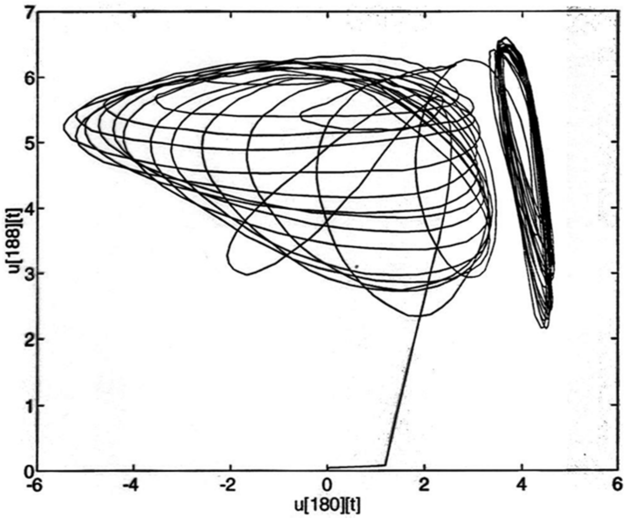
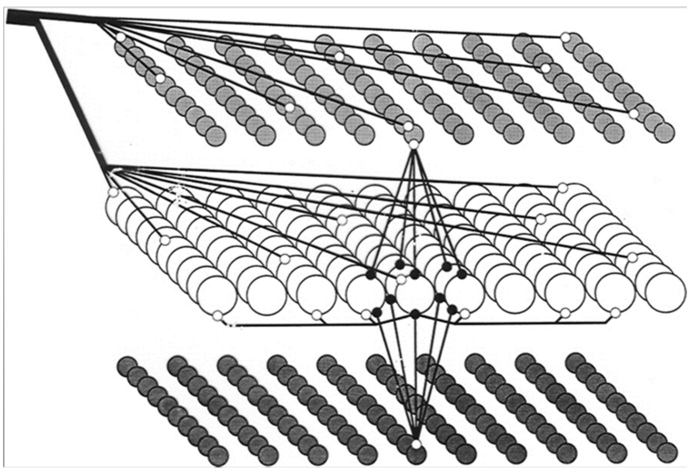
Continuous learning and recall



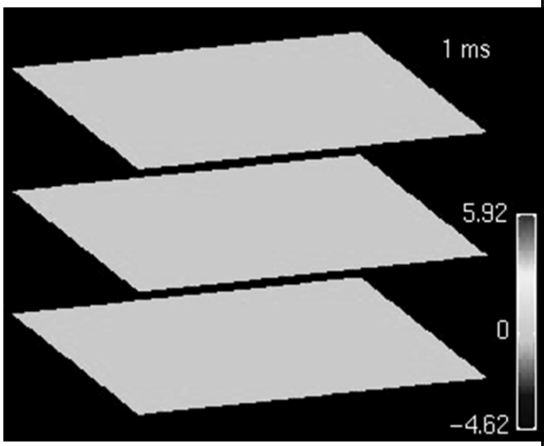
(Liljenström, H.., *Int. J. Intell. Syst*. 1995)

Neural network model of hippocampus

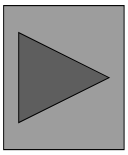


Simulation of neurodynamics

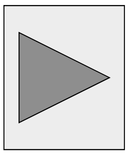
- A short pulse originating in one network corner
- Results in complex network dynamics
- Due to network connectivity (excitatory and inhibitory connections)



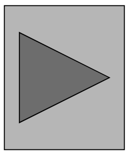
Simulations with neural networks



Fish swimming



4-legged robot



4-legged robot 2

Complexity out of simplicity

As we have seen in previous lectures, complex patterns or behaviours can arise in simple systems (e.g. the double pendulum) or from simple rules (e.g. randomn walks in triangles).

Another example of this is the so called ”Game of Life”, first studied by Princeton professor John Conway.

Game of Life

The Game of Life (or simply Life) is not a game in the conventional sense. There are no players, and no winning or losing. Once the "pieces" are placed in the starting position, the rules determine everything that happens later. Nevertheless, Life is full of surprises! In most cases, it is impossible to look at a starting position (or *pattern*) and see what will happen in the future. The only way to find out is to follow the rules of the game.

Game of Life

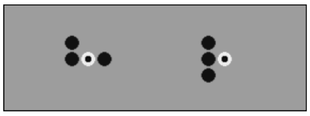
Rules of the Game of Life

Life is played on a grid of square cells--like a chess board but extending infinitely in every direction. A cell can be ***live*** or ***dead***. A live cell is shown by putting a marker on its square. A dead cell is shown by leaving the square empty. Each cell in the grid has a neighborhood consisting of the eight cells in every direction including diagonals.

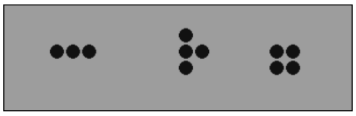
To apply one step of the rules, we count the number of live neighbors for each cell. What happens next depends on this number.

Game of Life

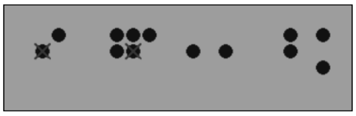
A dead cell with exactly three live neighbors becomes a live cell (birth).



A live cell with two or three live neighbors stays alive (survival).



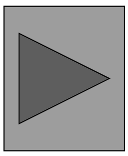
In all other cases, a cell dies or remains dead (overcrowding or loneliness).

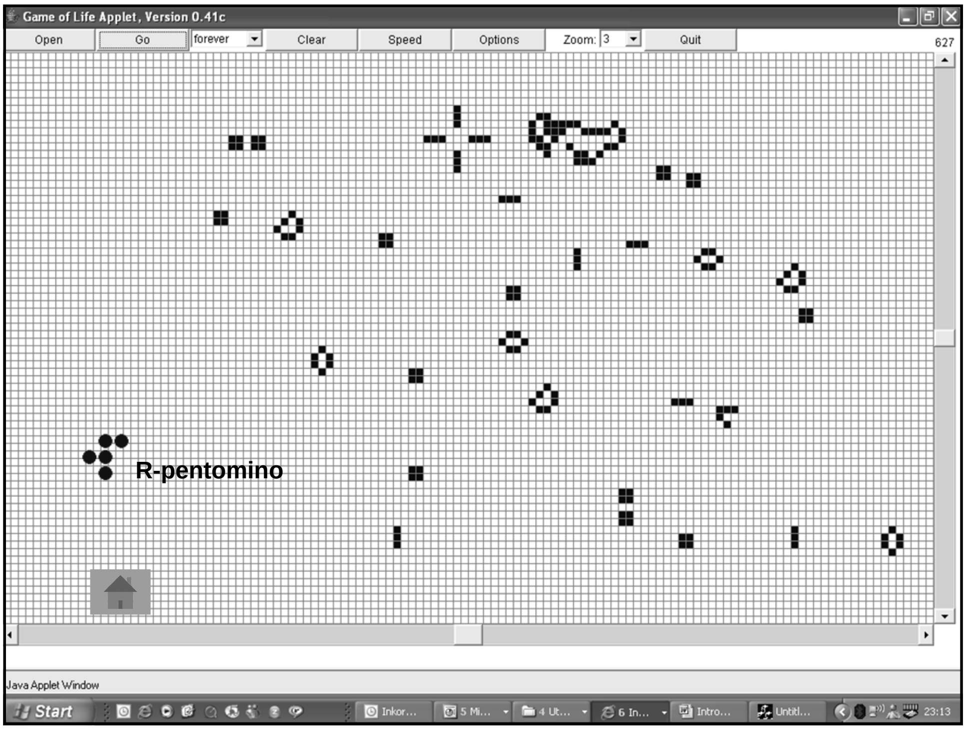


Game of Life

Note: The number of live neighbors is always based on the cells ***before*** the rule was applied. In other words, we must first find all of the cells that change before changing any of them.

Game of Life

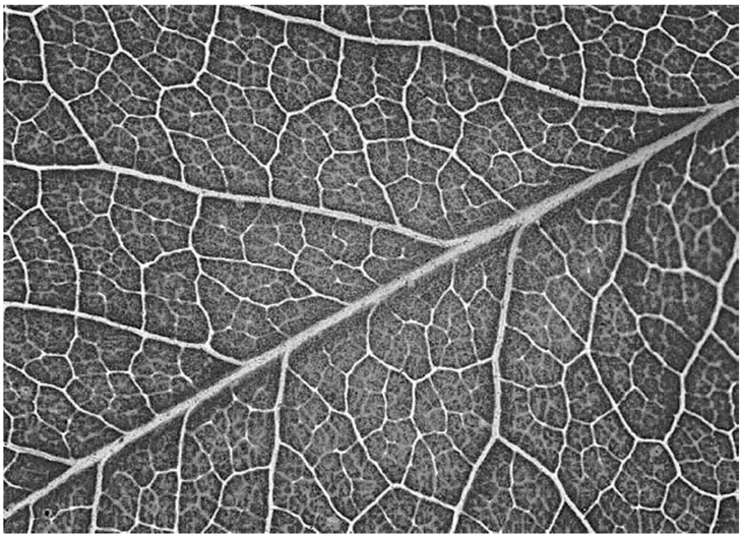




Fractals

What is it that impresses us about the branches of a tree, or the structure of a leaf?

How can we understand their complexity?



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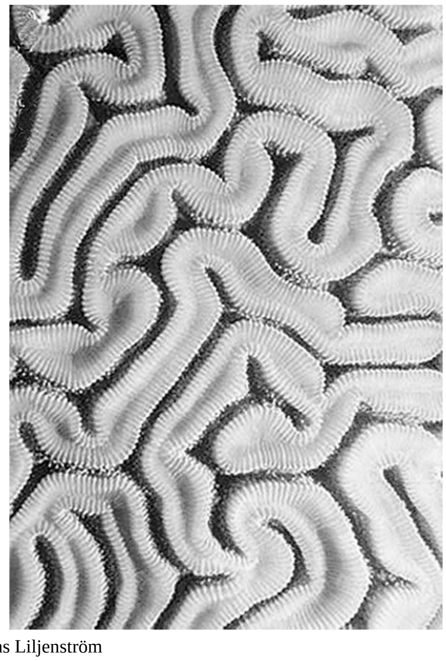

What are the reasons for the complexity of a coral (left) or a sponge (right)?

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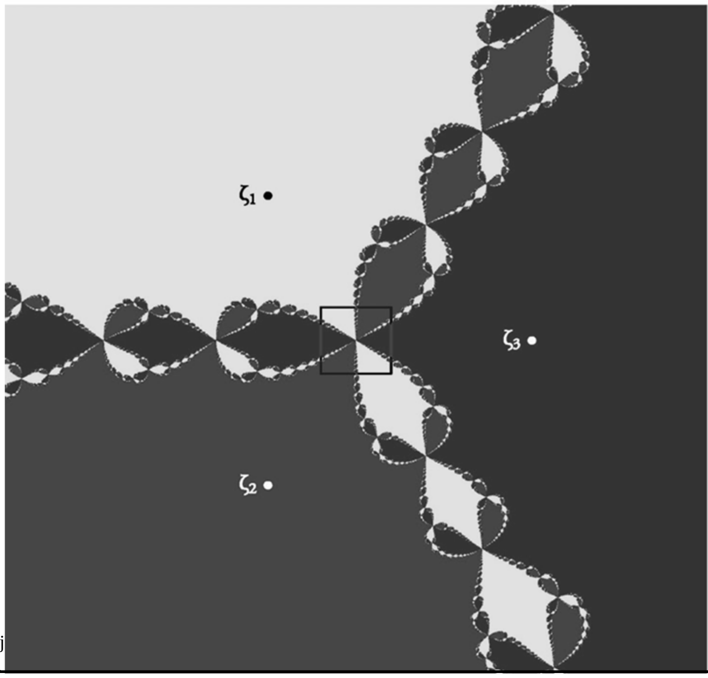
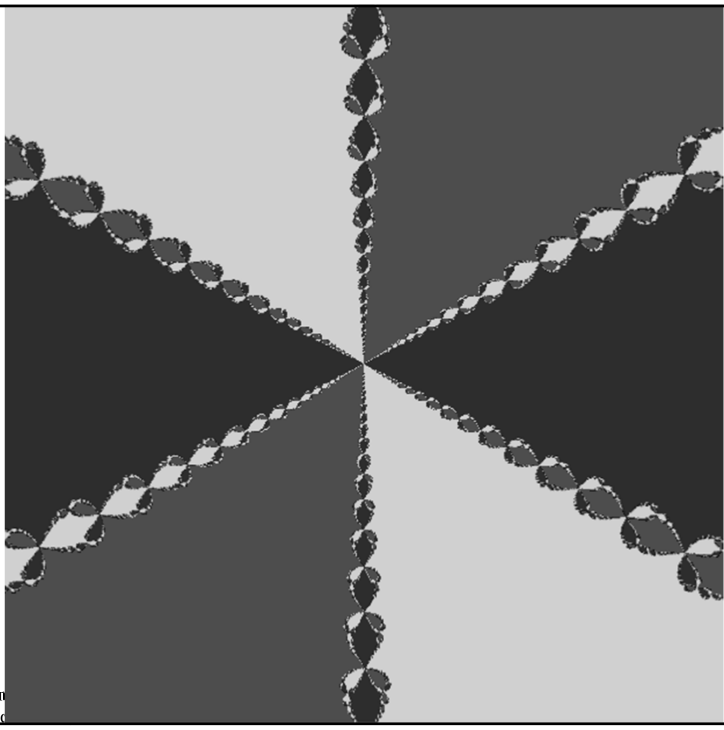


What do we see if we enlarge the center of the figure? ←



In this way we can "play God" and imitate nature by making pictures of real – looking plants like Barnsley's fern……

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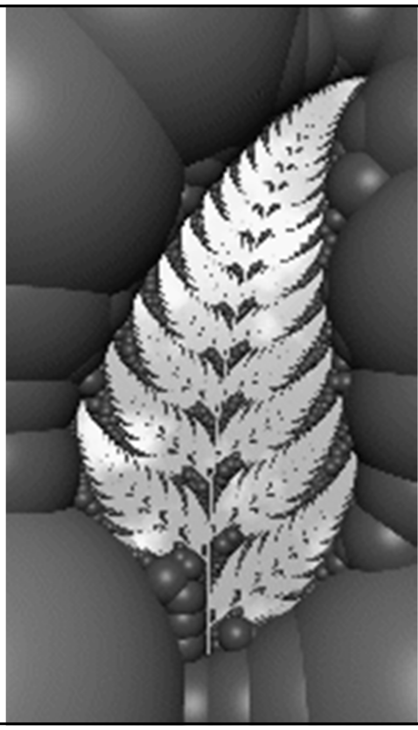

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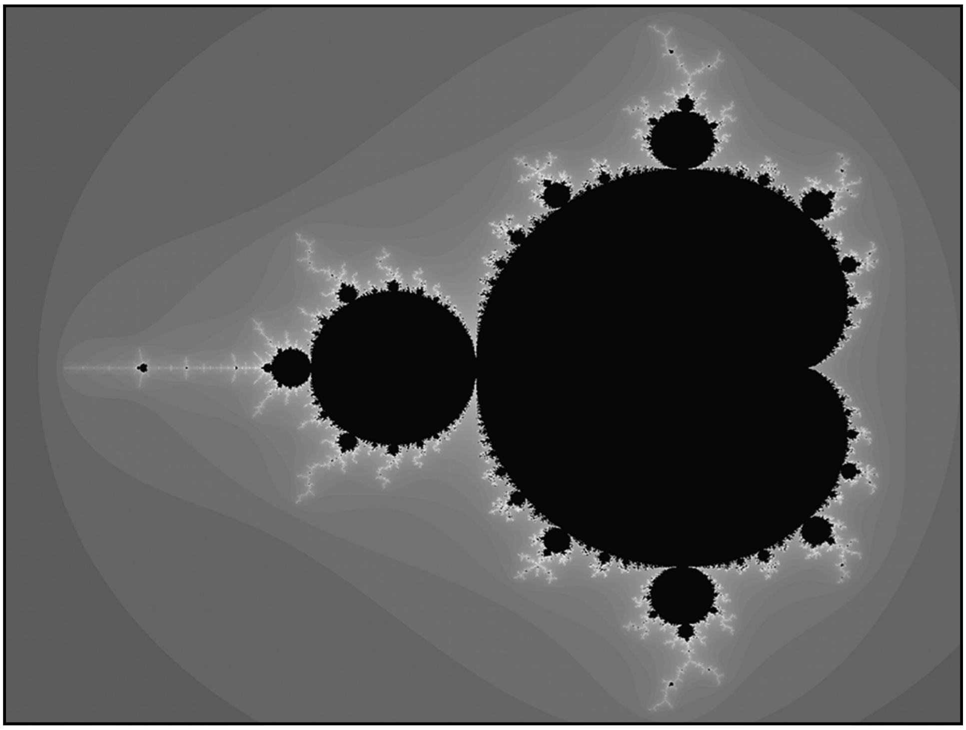
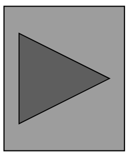


Fractals



www.youtube.com/watch?v=G_GBwuYuOOs

The Gaia hypothesis

The **Gaia hypothesis** is an ecological hypothesis proposing that the biosphere and the physical components of the Earth (atmosphere, cryosphere, hydrosphere and lithosphere) are closely integrated to form a complex interacting system that maintains the climatic and biogeochemical conditions on Earth in a preferred homeostasis.

Originally proposed by James Lovelock as the earth feedback hypothesis, it was named the Gaia Hypothesis after the Greek supreme goddess of Earth.

The hypothesis is frequently described as viewing the Earth as a single organism.

The Gaia hypothesis

James Lovelock defined Gaia as:

a complex entity involving the Earth's biosphere, atmosphere, oceans, and soil;

the totality constituting a feedback or cybernetic system which seeks an optimal physical and chemical environment for life on this planet.

The Gaia hypothesis

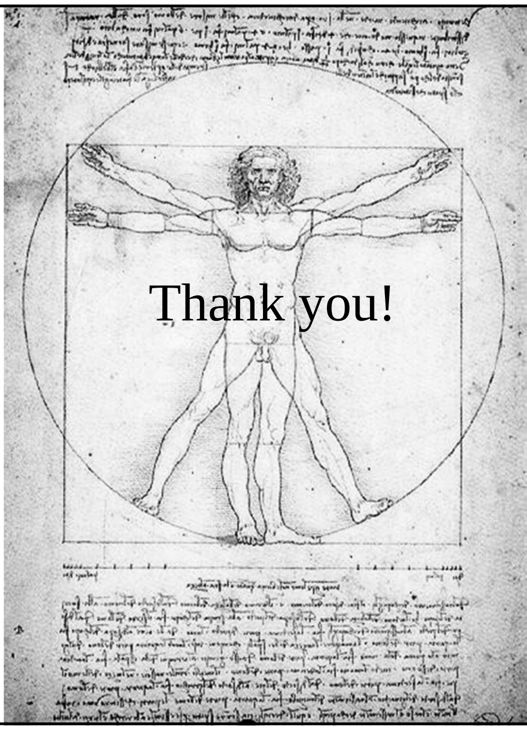
Lovelock suggested that life on Earth provides a cybernetic, homeostatic feedback system operated automatically and unconsciously by the biota, leading to broad stabilization of global temperature and chemical composition.

With his initial hypothesis, Lovelock claimed the existence of a global control system of surface temperature, atmosphere composition and ocean salinity. His arguments were:

– The global surface temperature of the Earth has remained constant, despite an increase in the energy provided by the Sun.
– Atmospheric composition remains constant, even though it should be unstable.
– Ocean salinity is constant.

The Daisyworld of the Gaia Theory

Forest Game model simulation



Thank you!